NAG Toolbox for MATLAB

f01qk

1 Purpose

f01qk returns the first ℓ rows of the real n by n orthogonal matrix P^{T}, where P is given as the product of Householder transformation matrices.

This function is intended for use following f01qj.

2 Syntax

```
[a, ifail] = f01qk(wheret, m, nrowp, a, zeta, 'n', n)
```

3 Description

P is assumed to be given by

$$P = P_m P_{m-1} \cdots P_1$$

where

$$P_k = I - u_k u_k^{\mathrm{T}},$$

$$u_k = \begin{pmatrix} w_k \\ \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar, w_k is a $(k-1)$ element vector and z_k is an $(n-m)$ element vector. w_k must be supplied in the kth row of **a** in elements $\mathbf{a}(k,1),\ldots,\mathbf{a}(k,k-1)$. z_k must be supplied in the kth row of **a** in elements $\mathbf{a}(k,m+1),\ldots,\mathbf{a}(k,n)$ and ζ_k must be supplied either in $\mathbf{a}(k,k)$ or in **zeta**(k), depending upon the parameter **wheret**.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **wheret – string**

Indicates where the elements of ζ are to be found.

wheret = 'I' (In **a**)

The elements of ζ are in **a**.

wheret = 'S' (Separate)

The elements of ζ are separate from **a**, in **zeta**.

Constraint: **wheret** must be one of 'I' or 'S'.

2: **m – int32 scalar**

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

3: **nrowp – int32 scalar**

ℓ, the required number of rows of P.

If $\mathbf{nrowp} = 0$, an immediate return is effected.

Constraint: $0 \leq \mathbf{nrowp} \leq \mathbf{n}$.

4: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m}, \mathbf{nrowp})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The leading m by m strictly lower triangular part of the array **a**, and the m by $(n - m)$ rectangular part of **a** with top left-hand corner at element $\mathbf{a}(1, \mathbf{m} + 1)$ must contain details of the matrix P. In addition, when $\mathbf{wheret} = \text{'I'}$, then the diagonal elements of **a** must contain the elements of ζ.

5: **zeta(∗) – double array**

Note: the dimension of the array **zeta** must be at least $\max(1, \mathbf{m})$ if $\mathbf{wheret} = \text{'S'}$, and at least 1 otherwise.

With $\mathbf{wheret} = \text{'S'}$, the array **zeta** must contain the elements of ζ. If $\mathbf{zeta}(k) = 0.0$ then P_k is assumed to be I, otherwise $\mathbf{zeta}(k)$ is assumed to contain ζ_k.

When $\mathbf{wheret} = \text{'I'}$, the array **zeta** is not referenced.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m}, \mathbf{nrowp})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The first **nrowp** rows of the array **a** are overwritten by the first **nrowp** rows of the n by n orthogonal matrix P^{T}.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, **wheret** $\neq$ 'I' or 'S',
or **m** < 0,
or **n** $<$ **m**,
or **nrowp** < 0 or **nrowp** $>$ **n**,
or **lda** $< \max(\mathbf{m}, \mathbf{nrowp})$.

7 Accuracy

The computed matrix P satisfies the relation

$$P = Q + E,$$

where Q is an exactly orthogonal matrix and

$$\|E\| \leq c\epsilon,$$

ϵ is the ***machine precision*** (see x02aj), c is a modest function of n, and $\|.\|$ denotes the spectral (two) norm. See also Section 7 of the document for f01qj.

8 Further Comments

The approximate number of floating-point operations is given by

$\frac{2}{3}m\{(3n-m)(2\ell-m) - m(\ell-m)\}$, if $\ell \geq m$, and

$\frac{2}{3}\ell^2(3n-\ell)$, if $\ell < m$.

9 Example

```
wheret = 'Separate';
m = int32(3);
nrowp = int32(5);
a    =    [-3.144584553986008,    -1.070546935661071,    -2.228344058124622,
0.6332849259420066, 0.7619486847328875;
          0.5276878374877219,  -2.834516318284424,  -2.228344058124622,  -
0.1662468409100804, 0.0945215703740671;
            0.3765535922552591,   0.3765535922552591,   -5.385164807134504,
0.07531071845105182, -0.4368021670161005;
     0, 0, 0, 0, 0;
     0, 0, 0, 0, 0];
zeta = [1.009150337862724;
     1.298065178174413;
     1.232861284531129];
[aOut, ifail] = f01qk(wheret, m, nrowp, a, zeta)
```

```
aOut =
   -0.1310   -0.1310   -0.3276   -0.6551   -0.6551
   -0.5170   -0.5170    0.5499    0.2494   -0.3175
   -0.4642   -0.4642   -0.5199   -0.0928    0.5385
   -0.5054    0.5054   -0.3957    0.4946   -0.2967
   -0.4946    0.4946    0.4043   -0.5054    0.3032
ifail =
           0
```